

Time-frequency characteristics analysis of free-field pressure with underwater explosion

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Abstract: To obtain the characteristics of the free-field pressure with underwater explosion, the time-frequency characteristics of the monitored free-field pressure signals are studied through wavelet analysis, based on the experimental free-field pressure data subjected to underwater explosion. By using these signals, the pressure-time curves and the energy distributions in different blasting frequency bands are obtained. The result shows that the obtained time-frequency characteristics of free-field pressure can be used to generate the shock wave signal, the after flow signal, and the second pressure pulse, and by analyzing the energy and frequency component, it is easy to get the detailed time-frequency information of the free-field pressure signal intensity, frequency and duration. More than 90% of the energy of the shock wave pressure signals condenses in the band lower than 8 kHz, and the highest level of energy appears below the frequency of 4 kHz, and special attention should be directed to the low frequency band below 250 Hz of the after flow signal as well as the second pressure pulse for ship structure and equipment. Overall, this paper provides reference for shock-resistance analysis of the ship structure.

Key words: underwater explosion; wavelet analysis; time-frequency characteristics; free-field pressure

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0 Introduction

In the research on anti-explosion and anti-shock of the ship structure, free-field pressure is not only the important data to reflect external load of the ship with underwater explosion, but also a significant input parameter in the calculation of anti-shock value^[1-2]. To improve the anti-explosion and anti-shock capability of the ship, a lot of ships and models with underwater explosion tests have been carried out in countries all over the world^[3-5]. In recent years, China has also carried out a lot of relevant tests, obtaining a number of test data subjected to pressure load^[6-7]. Meanwhile, some scholars have analyzed the influence of shock wave and bubble pulse pressure on ship structure and equipment^[8].

In the field of signal analysis, wavelet analysis is a

new analysis method put forward and developed in recent decades. The appearance of wavelet transform made up the deficiency of traditional Fourier analysis method. For example, window size of wavelet transform not only adapts to signal, but also provides localization information of time domain and frequency domain of non stationary signals. Therefore, wavelet analysis is known as "mathematical microscope" of signal analysis. Up to now, this method has been successfully applied in some signal time-frequency processing, and obtained some achievements in the analysis of non-stationary and nonlinear signals such as underwater explosion signal and engineering blasting signal^[9-11]. Free-field pressure signals with underwater explosion have such characteristics: non-stationary, transient and short time and so on. So, spectrum structure of the signal is changeable.

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Based on the characteristics of free-field pressure signal with underwater explosion, aiming at the measured pressure signal of underwater explosion, this study analyzes the energy distribution characteristics of the measured pressure signal by taking the advantages of multi-layer and multi-resolution of wavelet analysis, to obtain time-frequency characteristic information of free-field pressure signals in underwater explosive.

1 Basic principle

1.1 Wavelet analysis

Arbitrary signal $f(t) \in L^2(R)$ is set with $L^2(R)$ as energy-limited signal space, and $f(t)$ is called energy-limited signal^[12-13], namely:

$$f(t) \in L^2(R) \Leftrightarrow \int_R |f(t)|^2 dt < +\infty \quad (1)$$

If $\psi(t) \in L^2(R)$, then its Fourier transform $\hat{\psi}(\omega)$ satisfies admissible condition:

$$C_\psi = \int_{-\infty}^{+\infty} |\omega|^{-1} \left| \hat{\psi}(\omega) \right|^2 d\omega < \infty \quad (2)$$

Namely, C_ψ is bounded, then ψ is called a basic wavelet or mother wavelet. After basic wavelet is expanded and translated, a wavelet sequence can be obtained:

$$\psi_{a,b}(t) = |a|^{-1/2} \psi\left(\frac{t-b}{a}\right) \quad (3)$$

where $a, b \in R$ and $a \neq 0$. Among them, a is stretch factor, b is shift factor. Eq. is as following:

$$(W_\psi f)(a, b) = \langle f, \psi_{a,b} \rangle = |a|^{-1/2} \int_{-\infty}^{+\infty} f(t) \overline{\psi\left(\frac{t-b}{a}\right)} dt \quad (4)$$

Eq. (4) is continuous wavelet transform in regard to basic wavelet. Of which, $\overline{\psi\left(\frac{t-b}{a}\right)}$ is conjugate operation of $\psi\left(\frac{t-b}{a}\right)$. Thus, it can be seen that wavelet transform can transform one-dimensional signal into two-dimensional signal. From the above equation, it can be noted that the transformed function is two-dimensional, which can lay foundation for analyzing the time-frequency characteristics of signal. For practical calculation, wavelet transform is usually discretized, that is, discretize parameter a or b , or a and b at the same time, so that the discrete wavelet transform can be got. Generally taken:

$$a = a_0^m, b = nb_0 a_0^m; m, n \in Z$$

Taking $a_0 = 2, b_0 = 1$, dyadic wavelet can be gained as follows:

$$\psi_{m,n}(t) = 2^{-m/2} \psi(2^{-m}t - n) \quad (5)$$

where m is discrete sampling points of the signal. When m increases 1, the stretch factor a will be doubled, and the corresponding signal frequency is reduced by half, which is not only suitable for efficient calculation, but also easy to analyze.

1.2 Energy statistic on every frequency bands after wavelet transform

After the original response signal decomposed by wavelet, it can get the wavelet component in each analysis frequency band from high to low, and each wavelet component is still the time process signal. If the signal is analyzed to the n^{th} layer, the corresponding energy of each layer is:

$$E_i = \int |f_i(t)|^2 dt = \sum_{j=1}^m |y_{ij}|^2 \quad (6)$$

$$E = \sum_{i=1}^{n+1} E_i \quad (7)$$

where E_i is the energy sum corresponding to the i^{th} band signal. $f_i(t)$ represents wavelet decomposition of the i^{th} frequency band signal. y_{ij} refers to the discrete point amplitude of the n^{th} frequency band signal $f_i(t)$. E denotes the total energy of the analytical signal. $i = 1, 2, \dots, n+1; j = 1, 2, \dots, m$.

The proportion of each wavelet decomposition energy in the total energy of the analytical signal is:

$$k_i = \frac{E_i}{E} \quad (8)$$

2 Test of free-field pressure with underwater explosion

Test of free-field pressure with underwater explosion is performed in a large underwater explosion test pool, with the pool diameter of 50 m and water depth of 20 m. Due to the similarity of analytical methods, the following content only chooses one of the operating conditions to analyze a series of performed underwater explosion tests. Test conditions: the mass and water depth of TNT charge is 0.5 kg and 10 m respectively. The measuring point is the same as the water depth, and the distance between the measuring point and charge is 5 m. Fig. 1 shows free-field pressure time curves of underwater explosion experiment.

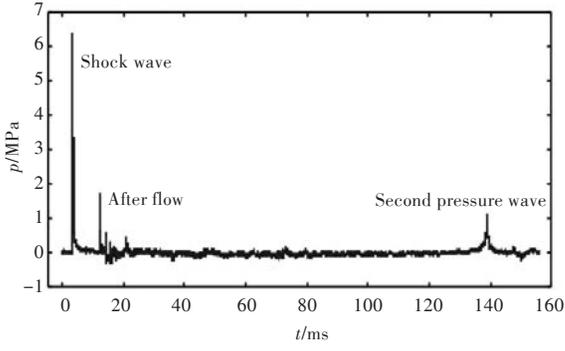


Fig.1 Free-field pressure curves of underwater explosion experiment

In relation to underwater explosion theory, the underwater explosion process is divided into 3 stages: 1) Shock wave pressure stage; 2) The after flow signal (diffusive flow) pressure stage is formed by radial movement of the water particle due to the velocity of bubble expansion less than the speed of sound in water; 3) Owing to bubble pulse, the second pressure wave is formed. Hence, the process of each stage is clearly measured. It is generally believed that only one pressure wave generated by bubble pulse is meaningful, so only the second pressure pulse is analyzed and the subsequent bubble pulse is not analyzed. In accordance with the related study of underwater explosion^[1], the pressure wave of each stage has a good estimation formula:

$$p(t) = p_m \cdot e^{-t/\theta} \quad (9)$$

$$p_m = 53.3(W^{1/3}/R)^{1.13} \quad (10)$$

$$\theta = 10^{-4}W^{1/3}(W^{1/3}/R)^{-0.24} \quad (11)$$

$$T = 2.11W^{1/3}/(H + 10.3)^{5/6} \quad (12)$$

where p_m represents peak pressure of shock wave signal. θ is exponential decay time constant. T refers to bubble pulse period. W stands for the mass of TNT charge. R means the distance between charges. H denotes the water depth of charges.

As can be seen from Fig. 1, the peak pressure of shock wave measured by the test is 6.41 MPa, and the peak pressure calculated by Eq. (10) is 6.66 MPa with error about 3.8%. Similarly, the bubble pulse period is 135.25 ms, and the bubble pulse period computed through Eq. (12) is 136.26 ms, with error about 0.74%. The experimental results are in agreement with the theoretical results, indicating that the test results are reliable.

3 Time-frequency characteristics analysis of free-field pressure

3.1 Signal wavelet decomposition

In the practical application of wavelet analysis, the selection of wavelet basis is a very important issue because using different wavelet basis for the same signal analysis will get different results^[14]. In the analysis non-stationary signal, Daubechies wavelet basis function is the most commonly used one as Daubechies wavelet series has good compact support, smoothness and approximate symmetry^[15]. In the Daubechies wavelet series, the db8 wavelet basis function is applied in non-stationary signal analysis. This study will also use the db8 wavelet basis function to decompose the measured signal.

The experimental set of the signal sampling frequency is 1 MHz, and the Nyquist frequency used for signal analysis is 0.5 MHz according to sampling theorem. Taking the measured free field pressure signal with underwater explosion as the object, the wavelet transform of test signal is carried out by using the wavelet basis function of db8 on the basis of the wavelet analysis theory. In this study, the observed signal is decomposed into 11 layers, and the decomposition frequency band is shown in Table 1. After decomposing, 12 coefficients of wavelet decomposition are obtained.

Table 1 Corresponding frequency band of wavelet decomposition

Wavelet coefficient	Frequency range/Hz	Wavelet coefficient	Frequency range/Hz
d1	250 000~500 000	d7	3 906.25~7 812.5
d2	125 000~250 000	d8	1 953.125~3 906.25
d3	62 500~125 000	d9	976.56~1 953.125
d4	31 250~62 500	d10	488.28~976.56
d5	15 625~31 250	d11	244.14~488.28
d6	7 812.5~15 625	a11	0~244.14

After the free-field pressure signal is decomposed by wavelet, to determine whether the decomposed signal can truly reflect the measured pressure signal, the decomposed signals are completely reconstructed and compared with the measured signal for calculating the relative error. The relative error distribution

of the reconstructed signal and the original signal is shown in Fig. 2, which indicates that the relative error between the reconstructed signal and the measured signal is of little difference less than 10^{-11} level. In addition, Fig. 2 also demonstrates that it is appropriate to decompose the pressure signal of the underwater explosion by utilizing the selected wavelet basis. The lost energy in the process of signal decomposition is negligible and can truly reflect the condition of the test signal. As can be noted from Fig. 3, reconstructed signals of free-field pressure in undergoing wavelet decomposition at different levels is based on db8 wavelet basis, respectively corresponding to the 12 frequency bands in Table 1. In Fig. 3, a11 stands for low frequency component and d1–d11 are high frequency components.

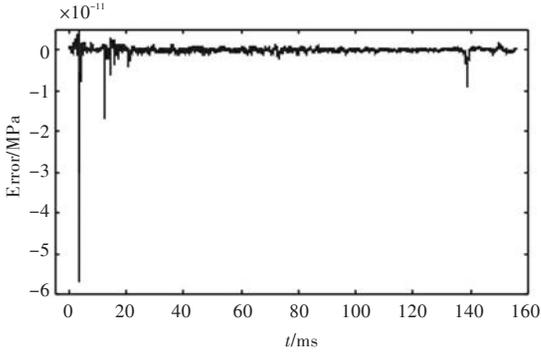
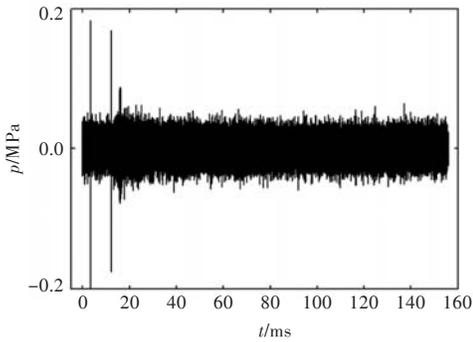
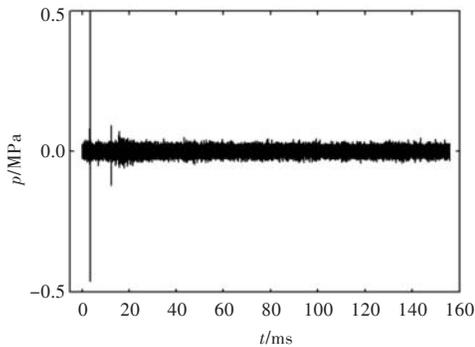


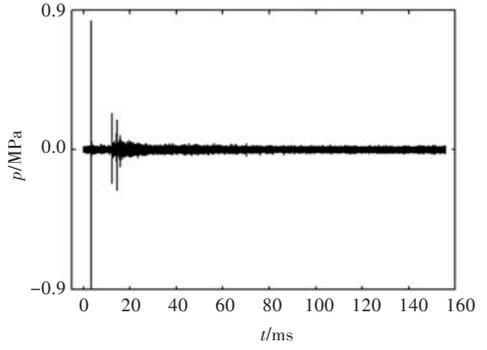
Fig.2 Distribution of relative error



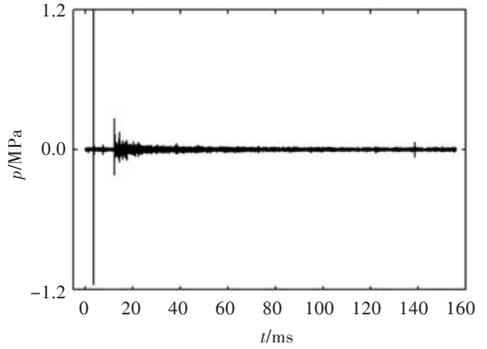
(a) $i=d1$



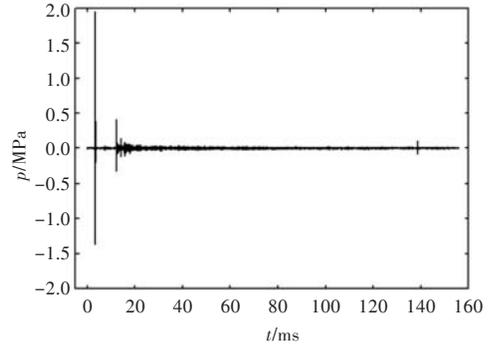
(b) $i=d2$



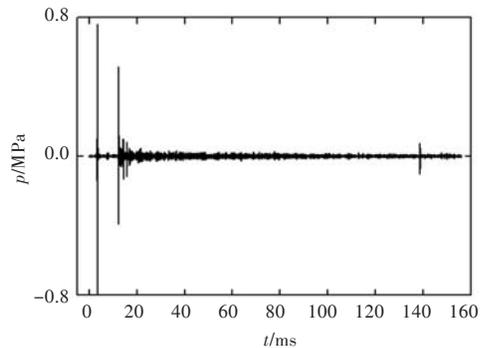
(c) $i=d3$



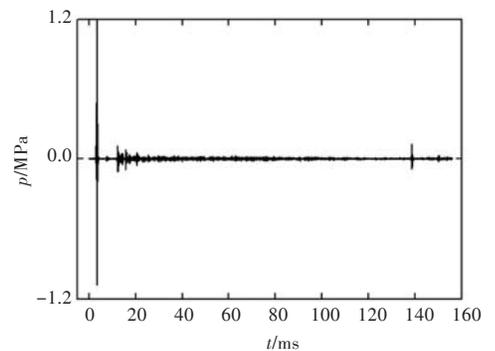
(d) $i=d4$



(e) $i=d5$



(f) $i=d6$



(g) $i=d7$

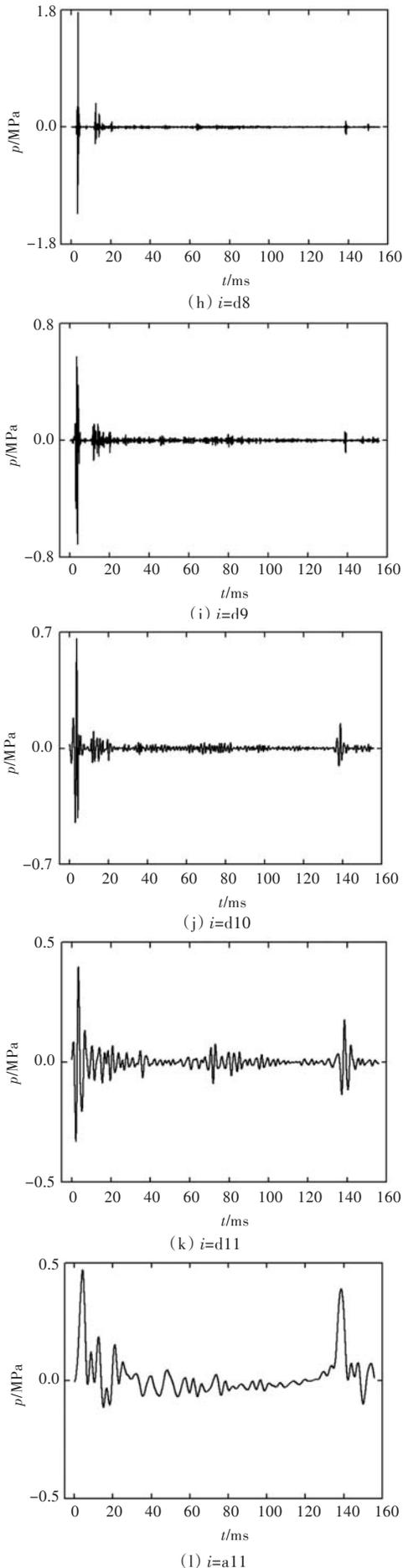


Fig.3 Reconstructed signals of free-field pressure in undergoing wavelet decomposition at different levels

3.2 Energy distribution statistics of each frequency band

In order to analyze the energy distribution of the free-field pressure signal with underwater explosion during the period of analysis, the distribution of signal relative energy in different frequency bands (f_i), the proportion of the signal energy in the total energy k_i as well as the signal pressure amplitude of each band $P_{i\max}$ can be obtained by using Eq. (6)–Eq. (8) according to the layered reconstruction signal of wavelet transform. The band parameters for free-field pressure is shown in Table 2.

Table 2 Band parameters for free-field pressure

i	f_i/Hz	$P_{i\max}/\text{MPa}$	$k_i/\%$
d1	250 000~500 000	0.18	0.56
d2	125 000~250 000	0.50	0.96
d3	62 500~125 000	0.83	0.63
d4	31 250~62 500	1.33	1.16
d5	15 625~31 250	1.95	3.0
d6	7 812.5~15 625	0.76	2.47
d7	3 906.25~7 812.5	1.20	4.87
d8	1 953.125~3 906.25	1.77	16.47
d9	976.56~1 953.125	0.57	8.53
d10	488.28~976.56	0.66	10.91
d11	244.14~488.8	0.40	11.25
a11	0~244.14	0.47	39.19

To further analyze the energy distribution of free-field pressure with underwater explosion at different stages and different frequency bands, the time history of free-field pressure wave is divided into 3 stages on account of underwater explosion theory: the shock wave stage, the after flow stage, and the second pressure wave stage. The midpoint of the peak pressure time of each stage is defined as the dividing line, that is, the shock wave pressure and the after flow pressure are divided by the midpoint of the shock wave pressure and the peak pressure time of the after flow signal. Besides, the after flow signal and the second pressure pulse are divided by the midpoint of the shock wave and the peak time of the second pressure pulse (the maximum point of bubble pulse radius). Namely, 0–7.917 ms is the acting time of shock wave, 7.918–71.067 ms stands for the acting time of the after flow signal, and after 71.068 ms means the acting time of the second pressure pulse. According to the layered reconstruction signal of

wavelet transform, the energy of the shock wave signal, the after flow signal and the second pressure pulse at different stages and different frequency band are counted. Eventually, energy ratio for each stage is shown in Fig. 4 and energy ratio for each spectrum in each stage is shown in Fig. 5.

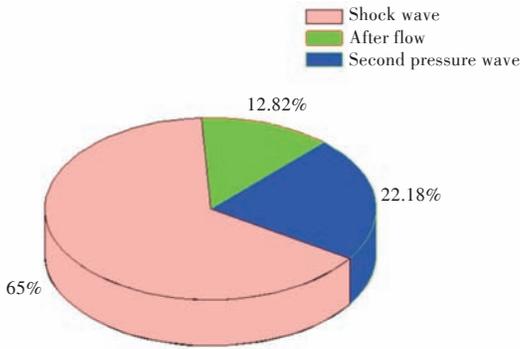


Fig.4 Energy ratio for each stage

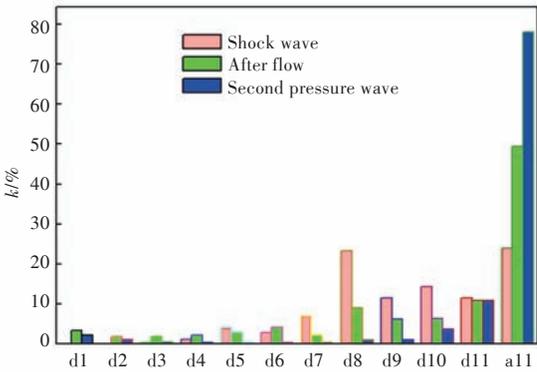


Fig.5 Energy ratio for each spectrum in each stage

3.3 Result analysis

1) After the load signal of free-field pressure with underwater explosion is decomposed by wavelet, the features like multivariate analysis and multi-resolution of wavelet analysis are adopted to decompose the measured pressure signal into specific information, that is, 12 frequency band time-pressure signal. The energy distribution feature of the free-field pressure signal is obtained by energy statistics of the local information of each frequency band. After that, the magnitude of the impact energy contained in each frequency band can be used to characterize the intensity information of the impact signal in the respective frequency bands. It can be observed from Table 2 that the energy of pressure signal in the low frequency band a11 (0–244.14 Hz) is the largest in each frequency band, reaching 39.19%. The energy of pressure signal in the frequency band d7–a11 (0–7812.5 Hz) takes up a great ratio, accounting for

91.22% of the total energy. Thus, it is clear that the energy of free-field pressure signal is mainly below 8 kHz, especially below 4 kHz. In the high frequency bands (125–500 kHz) d1 and d2, not only the energy ratio of pressure signal is very small, but also the pressure amplitude is very small. Then, combined with the pressure-time curves of d1 and d2, it can be found that its signal-noise ratio is very low. It can be assumed that the energy of two bands is mainly from noise. Thus, the energy information of each frequency band can reflect the impact strength of impact pressure in the corresponding frequency band after the free-field pressure signal with underwater explosion being transformed by wavelet.

2) It can be seen from Fig. 3 that the free-field pressure signal contains abundant frequency components. The frequency band is very wide and the pressure fluctuation is from high frequency to low frequency. Combined with the energy distribution, the main frequency band of free field pressure load can be got. Consequently, the effective frequency information of free-field pressure signal with underwater explosion can be characterized by the statistical information of the energy at different levels on each frequency band after being transformed by signal wavelet.

3) Through the analysis of Table 2, it can be noted that the energy peak of each wavelet band and peak impact pressure is not in the same frequency band. Subsequently, combined with Fig. 3, it can be found that if the pressure signal decays slowly in a wavelet band for a long time, the energy it contains will be relatively large. Further analysis shows that the maximum amplitude of pressure on all frequency bands after signal decomposition is d5(15 625–31 250 Hz), which is much smaller than that of a11 (0–244.14 Hz). By comparing the pressure-time curves of d5 and a11 wavelet frequency band, it can be observed that although the amplitude of d5 band is the largest, its impulse pressure signal decays very fast with short duration, so its relative energy is small. As a result, after wavelet analysis of the free-field pressure signal with underwater explosion, the wavelet layered reconstructed signal on each frequency band can reflect the attenuation and duration information of the pressure load in the corresponding frequency band.

4) Through the energy statistics of the pressure-time curves at different stages, Fig. 4 shows

that the shock wave signal, the after flow signal, and the second pressure wave all contain a lot of energy. The energy of shock wave signal is the largest stage, concentrated 65% energy. In combination with Fig. 3, it can be seen that the shock wave stage is the most wide frequency band with biggest impact strength. The frequency band contained in after flow signal is similar to the shock wave signal with relatively low energy. The second pressure wave signal contains relatively narrow band, mainly in the low frequency band with relatively concentrated energy band. Therefore, based on wavelet analysis and energy statistic method, the energy distribution information of the shock wave signal, the after flow signal and the second pressure wave at each stage in different frequency bands of free-field pressure signal with underwater explosion can be gained.

5) Based on the energy ratio (Fig. 5) for each spectrum in each stage of the shock wave signal, the after flow signal and the second pressure wave, it can be observed that the energy band of the shock wave signal is broad, but the main energy is concentrated in the frequency band that is below 4 kHz (accounting for 84.46%). The distribution energy of the after flow signal is similar to that of the shock wave signal, but a large part of the energy is concentrated in the low frequency band. Below 244.14 Hz, 50% of the energy is concentrated here. The second pressure wave is completely different because its energy is relatively concentrated with strong low-frequency characteristics. Below 244.14 Hz, its energy accounts up to 78%. Therefore, by means of wavelet analysis, the energy frequency band distribution of shock wave is wider, and the energy of bubble pulse pressure is dominated by low frequency. The instantaneous impact pressure peak is large during underwater explosion and energy distribution frequency range is wide at the moment. That is the main cause of the local damage of the ship structure when the ship is subjected to underwater explosion. Meanwhile, the low frequency band with large energy is easy to couple with the overall vibration mode of the ship. It is the main threat to the total strength of the hull. What's more, the low frequency band coincides with the installation frequency of many ship equipment, which is easy to produce resonance phenomenon with the isolation system of the equipment, resulting in dam-

age to the equipment.

4 Conclusions

In summary, the following conclusions can be drawn:

1) Compared with traditional Fourier analysis method, wavelet analysis method has better time-frequency analysis capability, which can meet the requirements of time-frequency characteristic analysis of free-field pressure load signal with underwater explosion, and can get the pressure-time curves of the signal under different frequency bands as well as detailed information like signal intensity, frequency and duration.

2) Based on wavelet analysis and energy statistic method, the energy distribution information of the shock wave signal, the after flow signal and the second pressure wave from free-field pressure load signal in different frequency bands can be obtained. The results show that the concentration energy of shock wave in the frequency band below 8 kHz is more than 90%. Of which the energy contained in the band below 4 kHz is the largest, and the after flow signal, and the second pressure wave contain much energy in the low frequency range below 250 Hz, which will have an impact on the hull structure and equipment. Hence, special attention should be paid.

3) Wavelet analysis method provides a new way to study the impact load with underwater explosion and the failure mechanism of ship structure. As a result, the results of correlation analysis have practical reference value for the design of ship structure and equipment.

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水下爆炸条件下自由场压力载荷时频特征分析

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摘要: 在舰船结构水下爆炸试验中, 为了研究水下爆炸条件下水中自由场压力载荷的时频特征, 针对某水下爆炸试验自由场压力测试试验数据, 基于小波分析对信号进行时频特性分析, 得到水中自由场压力信号的时频分布和能量分布状况。分析结果表明: 针对水下爆炸自由场压力载荷, 基于小波分析技术对其时频特征进行分析, 可得到水下爆炸自由场压力载荷所包含的频率信息、强度信息以及不同频段下的载荷持续作用时间等信息; 另外, 可对冲击波、滞后流和二次压力波这3个不同信号阶段进行频段与能量统计分析; 在不同频段上对冲南击信号的能量进行统计发现, 冲击波阶段在8 kHz以下频段集中了超过90%的能量, 其中4 kHz以下频段的能量最大, 在滞后流和二次压力波阶段, 需特别重视250 Hz以下的低频段对船体结构及设备的影响, 该结果对舰船结构及设备的抗冲击防护具有借鉴意义。

关键词: 水下爆炸; 小波分析; 时频特征; 自由场压力